CSS322: Scientific Computing
Midterm Mock Exam

curated by The Peanuts

Name.............ID.............Section.............Seat No.............

Conditions: Semi-closed Book

Directions:

1. This exam has 11 pages (including this page).
2. You may bring **two A4 cheat sheets**, written/printed/photocopied on both sides. They must be submitted with your exam paper. Treasure them well, for they will not return.
3. Calculators are allowed. Dictionaries are not.
4. Cheating is strictly prohibited.
5. For inspiration you may look at the ceiling, but not at your friend
6. Good luck!

*For solution, **click here**.*
Will be available soon, probably the night before the real exam.

Question 1

(a) Let $v = \begin{bmatrix} -3 \\ 2 \\ 0 \\ -5 \end{bmatrix}$. Compute $\|v\|_1$, $\|v\|_2$, and $\|v\|_\infty$.

(b) Let $A = \begin{bmatrix} 2 & -3 & 1 \\ 0 & 4 & -2 \\ -1 & 0 & 3 \end{bmatrix}$. Compute $\|A\|_F$, $\|A\|_1$, and $\|A\|_\infty$.

Question 2

Perform Gaussian Elimination with Partial Pivoting (GEPP) on the matrix

$$A = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 2 & 1 \\ 2 & -1 & 3 \end{bmatrix}$$

to find its $P^T LU$ factorization.

Question 3

We are solving the linear system

$$\begin{bmatrix} 4 & 1 \\ 1 & 3 \end{bmatrix} x = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

and have computed an approximate solution

$$x_0 = \begin{bmatrix} 1.1 \\ 1.9 \end{bmatrix}.$$

Perform one iteration of iterative refinement to obtain a better approximate solution x_1.

Question 4

Given the matrix $A = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$:

(a) Find A^{-1}.

(b) Compute the condition numbers $\mathrm{cond}_1(A)$ and $\mathrm{cond}_\infty(A)$.

Question 5

Find the Cholesky factorization (LL^T factorization) of the symmetric positive definite matrix by the algorithm described in the lecture:

$$A = \begin{bmatrix} 4 & -2 & 2 \\ -2 & 2 & -1 \\ 2 & -1 & 5 \end{bmatrix}$$

Show your calculations step by step.

Question 6

For what range of values of x is it difficult to compute

$$f(x) = \sqrt{x+1} - \sqrt{x}$$

accurately in floating-point arithmetic? Explain why.

Question 7

Find the Lagrange interpolant $p(t)$ for the following data points:

$$(0, 2), \quad (1, 3), \quad (3, 1), \quad (4, 5)$$

Also, evaluate $p(2)$.

Question 8

Suppose we have the following six data points:

$$(-1, 3), \quad (0, -2), \quad (1, 1), \quad (2, 5), \quad (3, -4), \quad (5, 8)$$

(a) Set up the linear system that needs to be solved in order to compute the polynomial interpolant in Newton basis for the above data points. Write the full matrix equation $Ax = b$. No need to solve the system.

(b) What algorithm should be used to solve this system and why? What is its computational complexity?

Question 9

Consider interpolating $f(t) = e^{-2t}$ with a polynomial of degree 2, using sample points at $t = 0, 0.5, 1$. Give a reasonably tight upper bound on the error of the interpolant over the interval $[0, 1]$. **Hint:** *The derivative of* e^{-cx} *is* $-ce^{-cx}$

Question 10

(a) Use the method of undetermined coefficients to derive a 3-point quadrature rule on the interval $[a, b]$ using

$$x_1 = a, \quad x_2 = \frac{a+b}{2}, \quad x_3 = b$$

Set up the system of equations and solve for the weights.

(b) Apply your quadrature rule from part (a) to approximate $\int_0^2 x^2 dx$.

(c) Compare your result with the exact value of the integral.