CSS321: Theory of Computation
Midterm Mock Exam

curated by The Peanuts

Name.............ID.............Section.............Seat No.............

Conditions: Open Book

Directions:

1. This exam has 16 pages (including this page).
2. You may use a calculator, but it won't help you prove languages are non-regular.
3. Dictionaries are not allowed. Neither is asking the Pumping Lemma for help (it's not here!).
4. Cheating is strictly prohibited.
5. Good luck! May all your states be accepting.

The solution will never be released, sorry!

Question 1

Consider the following statements:

(a) $\{a\} \in \{\{a\}, \{b\}\}$

(b) $\{a, b\} \subseteq \{\{a\}, \{b\}, a, b\}$

(c) $2^{\{a,b\}} = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

(d) For any sets A and B, if $A \subseteq B$ then $2^A \subseteq 2^B$

(e) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Which of the above statements are true? ________________________

Question 2

Let $A = \{1, 2, 3\}$ and $R = \{(1, 1), (1, 2), (2, 2), (2, 3), (3, 1), (3, 3)\}$ be a relation on A. Consider the following properties:

(a) R is reflexive

(b) R is symmetric

(c) R is transitive

(d) R is antisymmetric

(e) R is a partial order

Which of the above properties hold for R? ________________________

Question 3

Consider the following statements about regular languages:

(a) $L((a \cup b)^* a) = \{w \in \{a, b\}^* \mid w \text{ ends with } a\}$

(b) $L(a^* b^*) \cap L(b^* a^*) = \{a^n b^n \mid n \geq 0\}$

(c) For any regular language L, $L^* = L^+ \cup \{\varepsilon\}$

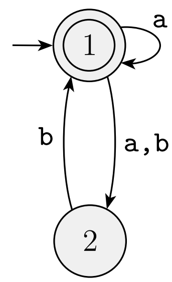
(d) The language $\{a^n b^m \mid n \neq m\}$ is regular

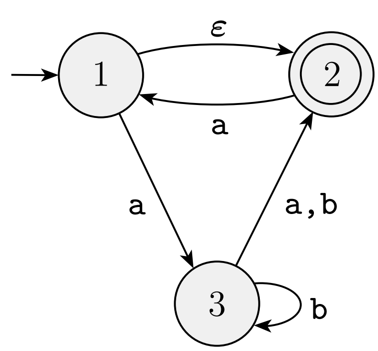
(e) Every finite language is regular

Which of the above statements are false? ______________

Question 4

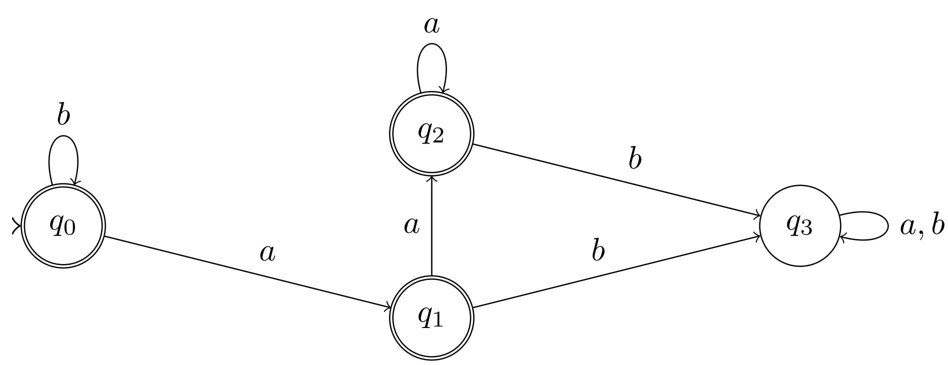
Convert the following two nondeterministic finite automata to equivalent deterministic finite automata.





Question 5

Consider the following finite automaton A over the alphabet $\Sigma = \{a, b, c\}$.



a) Is A deterministic? If not, convert A into a DFA.

b) Is A minimal? If not, convert A into a minimal DFA.

c) Convert A into a regular expression.

Question 6

Prove by induction that $n^3 + (n+1)^3 + (n+2)^3$ is divisible by 3 and $n > 0$

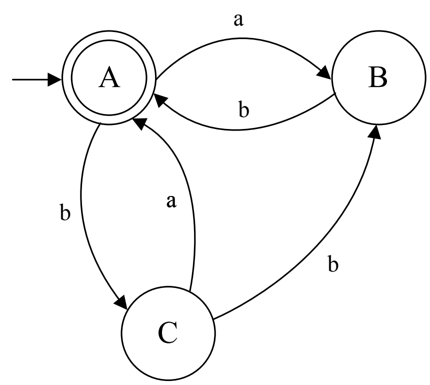
Question 7

Construct a DFA equivalent to the NFA $M = (\{a, b, c, d\}, \{0, 1\}, \delta, a, \{b, d\})$ where δ is given below and informally describe the language it accepts.

δ	0	1
a	$\{b, d\}$	$\{b\}$
b	$\{c\}$	$\{b, c\}$
c	$\{d\}$	$\{a\}$
d	$\emptyset$	$\{a\}$

Question 8

Find the regular expression for the following DFA.



Question 9

Show that $(\emptyset)^* = \epsilon$ for regular expression

Question 10

Let $\Sigma = \{a, b\}$ and let L_1 be the language over Σ given by the regular expression $(ab \cup ba)^*$. Design a DFA for L_1.

Let $\Sigma = \{a, b\}$ and let $L_2 = \{w \in \Sigma^* | \text{w does not contain bbb as a substring}\}$. Design a DFA for L_2 and write a regular expression.

Question 11

Consider the following statements about cardinality and functions:

(a) Every subset of a countably infinite set is finite or countably infinite

(b) There exists a bijection from $\mathbb{N}$ to $\mathbb{N} \times \mathbb{N}$

(c) The set $2^{\mathbb{N}}$ is countably infinite

(d) If $f : A \to B$ is one-to-one and $|A| = |B|$, then f is onto

(e) The diagonalization principle can prove that some infinite sets have different cardinalities

Which statements are true? ________________________

Question 12

Let $A = \{a, b, c\}$ and $R = \{(a, a), (a, b), (b, c), (c, a)\}$ be a relation on A.
a) Find the smallest reflexive relation R_1 containing R.

b) Find the smallest reflexive and transitive relation R_2 containing R.

c) Is R_2 an equivalence relation? If not, what would you need to add to make it one?

d) For the relation R, find:

1. The row set of a: $R_a = \{x \in A \mid (a, x) \in R\} =$ __________
2. The diagonal set: $D = \{x \in A \mid (x, x) \notin R\} =$ __________

Question 13

Construct DFAs for the following languages over $\Sigma = \{0, 1\}$:
a) $L_1 = \{w \mid |w| \bmod 3 = 0\}$ (strings whose length is divisible by 3)

b) $L_2 = \{w \mid w \text{ contains an even number of 1's}\}$

c) $L_3 = \{w \mid w \text{ ends with } 01\}$

Question 14

Prove the following set theory identity: $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$